Combining Simulation with Heuristics to solve Stochastic Routing and Scheduling Problems

Kombination von Simulation mit Heuristiken als Lösungsansatz für Stochastic Routing und Scheduling

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Abstract: Many real-world problems in the production and logistics business are NP-hard even in their deterministic representation, and actually also show stochastic behaviour, where even the mathematical description of the – frequently empirical – distributions is difficult or even impossible. Therefore, an approach is acquired that enables the search for valid and reasonably good solutions under representation of the stochastic system behaviour. A suitable approach is to combine heuristic optimization with simulation techniques. This paper discusses how Monte-Carlo simulation can be combined with heuristics and meta-heuristics in order to efficiently solve such stochastic combinatorial optimization problems. The application is illustrated with examples in two different fields, including logistics and transportation – e.g. vehicle routing problems and inventory problems – as well as manufacturing and production – e.g. scheduling problems.

1   Introduction

There is an emerging interest of introducing randomness into combinatorial optimization problems as a way of describing new real problems in which most of the information and data cannot be known beforehand. This tendency can be observed in Van Hentenryck and Bent (2010), which provides an interesting review of many traditional combinatorial problems with stochastic parameters. Thus, those authors studied Stochastic Scheduling, Stochastic Reservations and Stochastic Routing in order to make decisions on line, i.e., to re-optimize solutions when their initial conditions have changed and, therefore, are no longer optimal. This type of analysis has designed the so-called Online Vehicle Routing Problems, in which re-optimization is needed apart from a previous situation. This set of routing problems seems to be well analyzed with the use of stochastic hypothesis in their definitions (Bent and Van Hentenryck, 2007) providing more reality in their formulation. Another routing field in which randomness has also been developed is the resolution

of inventory routing problems where the product usage is stochastic (Hemmelmayr et al. 2010). Bianchi et al (2009) have written an interesting survey of the appropriate metaheuristics to solve a wide class of combinatorial optimization problems under uncertainty. The aforementioned survey is a good reference for obtaining an appropriate list of articles regarding the use of meta-heuristics for solving Stochastic Combinatorial Optimization Problems (SCOPs) in different application fields.

In this paper we discuss how Monte-Carlo simulation (MCS) can be combined with heuristics and meta-heuristics in order to efficiently solve SCOPs in order to give an overview on the recent progress in this research area. Examples from two application fields illustrate the application of this hybrid methodology.

The paper is structured as follows: Section 2 describes the main ideas behind the generic type of algorithms we propose by combining simulation with heuristics (Sim-heuristic algorithms). Section 3 provides an example of application to SCOPs in the field of logistics and transportation. Likewise, Section 4 reviews an example in the field of manufacturing and production. Finally, Section 5 summarizes the main conclusions of this work.

2   Basic Approach of Sim-heuristic Algorithms

As shown in Figure 1, a Sim-heuristic approach combines combinatorial-optimization methodologies – e.g. heuristics and/or meta-heuristics – with simulation methodologies – e.g. Monte-Carlo Simulation, Discrete Event Simulation (DES), agent-based approaches, etc. – in order to efficiently deal with the two components of a SCOP instance: the optimization nature of the problem and its stochastic nature. In general, this is a simulation-optimization (SimOpt) approach with the simulation as evaluation function of the optimization algorithm (cp. März et al. 2010), which is defined by the German VDI as "Category D" approach (VDI 2013). Some examples of Sim-heuristic applications to different fields can be found in the optimization-simulation literature (Juan et al., 2011a, Peruyero et al. 2011; Gonzalez et al. 2012; Caceres et al. 2012). Typically, given a SCOP instance, a heuristic/meta-heuristic algorithm is run in order to perform an oriented search inside the solution space. This iterative search process aims at finding the feasible solution with the best possible value in the search space, which is expected to be near to the actual optimum as well. During the iterative search process, the algorithm must deal with the stochastic nature of the SCOP instance. One natural way to do this is by taking advantage of the capabilities simulation methods offer to manage randomness. Of course, other approaches can also be used instead of simulation – e.g. dynamic programming, fuzzy logic, etc. However, under the presence of historical data on stochastic behaviour, simulation allows for developing models that are both accurate and flexible. Specifically, randomness can be modelled utilizing a best-fit probability distribution – including parameterization – without any additional assumptions or constraints. Thus, simulation is usually integrated with the heuristic/meta-heuristic approach and it frequently provides dynamic feedback to the searching process in order to improve the final outcome. In some sense, simulation allows for extending existing and highly efficient meta-heuristics – initially designed to cope with deterministic problems – so that they can also be employed to solve SCOPs.

Obviously, one major drawback of this approach is that the results are not expected to be optimal anymore, since Sim-heuristics are combining two approximate

methodologies. Nevertheless, real-life problems are complex enough and usually NP-hard even in their deterministic versions. Therefore, Sim-heuristics constitute a quite interesting alternative for most practical purposes, since they represent relatively simple and flexible methods which are able to provide near-optimal solutions to complex real-life problems in reasonable computing time.
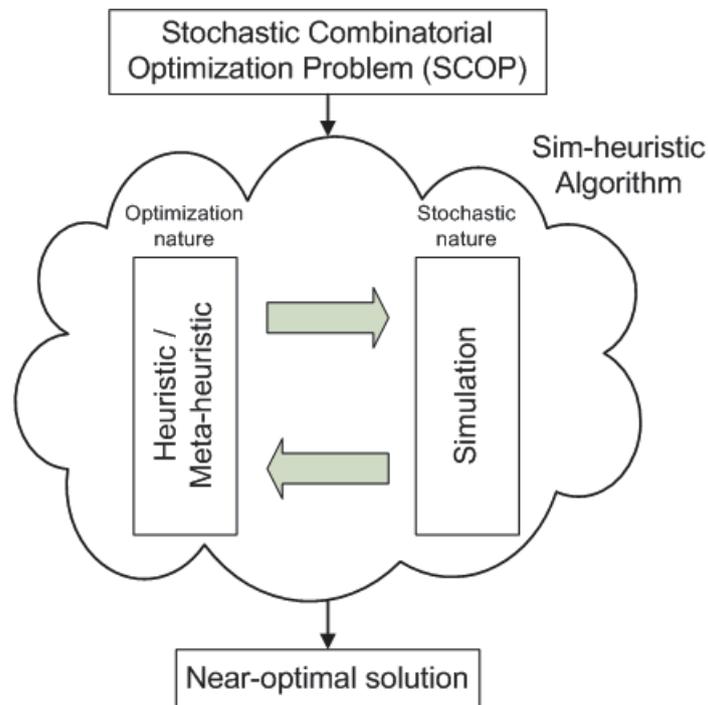

Figure 1: *Overview of the Sim-heuristic approach*

3   Case Example in Logistics and Transportation

The Inventory Routing Problem with stochastic demands (IRPSD) is an NP-hard problem that can be described as follows (Figure 2): consider a Capacitated Vehicle Routing Problem (CVRP) (Golden et al. 2008) with *n* nodes or retailing centers (RC), plus the depot. Each RC owns an inventory, which is managed by the central depot. For each RC, the inventory level at the end of a period depends on the initial stock level and also on the end-clients' demands during that period. These end-clients' demands are stochastic in nature. It will be assumed that, for each RC, it has been possible to use historical data to model end-clients' demands through a theoretical or empirical probability distribution. Notice that no particular assumption is made on the type of distribution used to model these demands. Therefore, at the end of each period there might be some costs associated with inventory holding and inventory stock-outs. These costs might be incorporated into the decision-making process and added to the distribution or routing costs, which are usually based on traveling distances and times. At the end of each period, inventory levels are registered by the

RC and updated in the central depot, so that a new routing strategy is defined for the new period taking into account the new data. The goal is to minimize total expected costs (distribution plus inventory-related costs) in each single-period scenario.
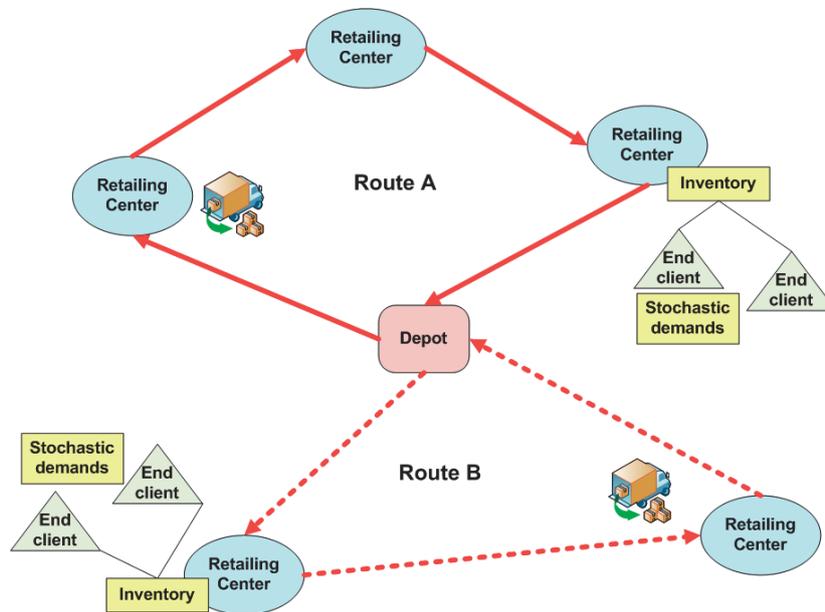

Figure 2: *General scheme of the IRP with stochastic demands*

When properly combined with heuristic techniques, MCS has proved to be extremely useful for solving different routing problems with stochastic demands (Juan et al. 2011a; Gonzalez et al. 2012). In the IRPSD context, Caceres et al. (2012) propose a hybrid approach which also combines MCS with an efficient CVRP heuristic. Specifically, given a customer, the following policies are considered:

1. no refill for that customer;
2. refill up to one quarter of its capacity;
3. refill up to half of its capacity;
4. refill up to three quarters of its capacity; and
5. full refill.

Thus, for each combination of customer-service policy, MCS is used to obtain estimates of the expected inventory costs associated with it, including both surplus and shortage costs. In the second step of the procedure, the authors consider the worst-case scenario from a distribution point of view, i.e., all customers receive a full refill. In this scenario, a fast heuristic is used to obtain a 'good' solution for the associated CVRP. This solution will provide an estimate of the total distribution costs under the full-refill policy. In the third step, they estimate for each customer the routing "marginal savings", i.e., the reduction in distribution costs associated with each non-full-refill policy. In order to do this, a fast heuristic is used again to solve a large set of CVRPs. Once these marginal costs have been estimated, for each customer an

approximated value for the total costs associated with each eligible policy can be obtained by simply adding up estimated routing and inventory costs. Therefore, for each customer, the associated eligible policies can be sorted from lower to higher total costs, thus defining a priority policy rank for each customer. In the fourth step, the 'top' policy for each customer (i.e., the one showing the lowest total cost) is selected, and a pseudo-optimal solution is obtained for the corresponding CVRP by using an efficient algorithm, e.g., the SR-GCWS-CS (Juan et al., 2011b). Finally, in the fifth step, a multi-start process is carried out. At each iteration of this multi-start process, a new policy is randomly selected for each customer and, in a similar way as in the previous step, a new pseudo-optimal solution is obtained for the corresponding CVRP. The best solution found so far is recorded.

After performing an extensive computational test, the authors show that their 'integrated' methodology outperforms the traditional sequential approach, in which each individual inventory level is optimized first and then the resulting vehicle routing problem is solved. Notice that their simulation-optimization approach can consider personalized refill policies for each customer, which contributes to significantly reduce the total costs over other approaches using standard refill policies. Figure 3 shows an example of the routing component of a solution to the IRPSD. In this solution, it can be observed that:

- the routing plan might not necessarily include all customers; and
- the refill policy can be different for each customer (different symbols represent different refill policies).
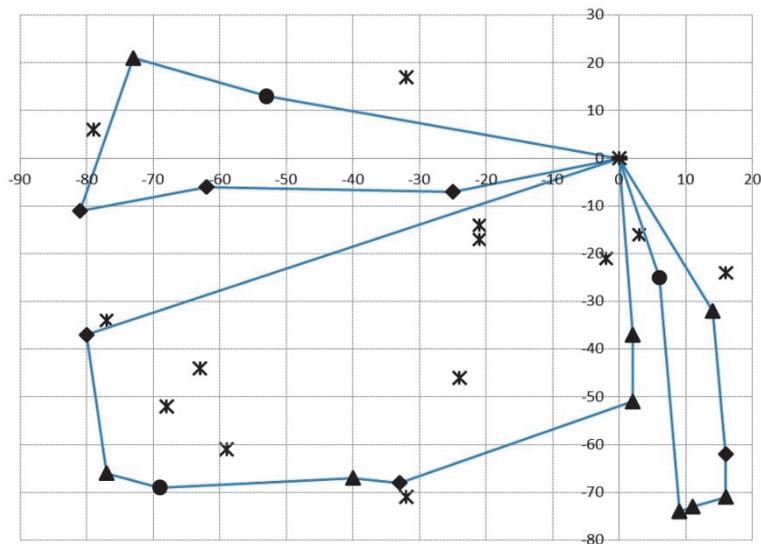

Figure 3: *Visual representation of the routing component of a solution*

4   Case Example in Manufacturing and Production

The Permutation Flow Shop Problem with Stochastic Times (PFSPST) can be seen as a generalization of the classical Permutation Flow Shop Problem (PFSP, Pinedo 1982)

in which the processing time of each job i in each machine j is not a constant value, but instead a random variable, P_{ij}, following a non-negative probability distribution – e.g. Log-Normal, Exponential, Weibull, Gamma, etc. Since uncertainty is present in most real-life processes and systems, considering random processing times represents a more realistic scenario than simply considering deterministic times. As a result, unforeseen circumstances can lead to sudden changes in the processing time of certain jobs in certain machines, which is likely to have noticeable effects on the predicted makespan – i.e., the total completing time (Figure 4). Therefore, one goal that can be considered when dealing with the PFSPST is to determine a sequence of jobs which minimizes the expected makespan or mean time to completion of all jobs. For these problems, simulation-optimization techniques and, in particular, the combination of simulation with meta-heuristics constitutes also a promising approach yet to be explored in its full potential. Figure 4 illustrates a simple PFSP with three jobs and three machines, where O_{ij} represents the operation time of job i in machine j ($1 \leq i \leq 3, 1 \leq j \leq 3$). Notice that, for a given permutation of jobs, even a single change in the processing time of one job in one machine (O_{21} in this case) can have a noticeable impact on the value of the final makespan.
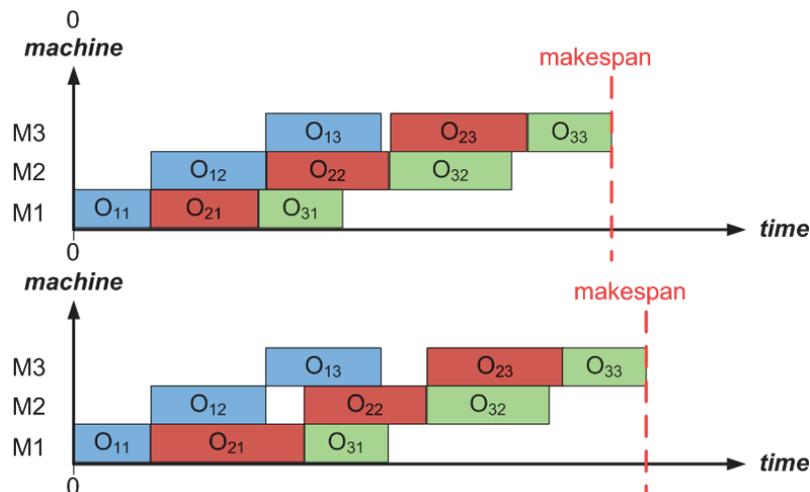

Figure 4: Variations in makespan due to stochastic processing times

As with other combinatorial optimization problems, a number of different approaches and methodologies have been developed to deal with the PFSP. These approaches range from complete optimization methods – such as linear and constraint programming – which can provide solutions to small-sized problems, to approximate methods – such as heuristics and meta-heuristics – which can provide near-optimal solutions for medium- and large-sized problems. Moreover, some of these methodologies are able to provide a set of alternative near-optimal solutions from which the decision-maker can choose according to his/her specific preferences. However, the situation with the PFSPST is quite different. In the articles by Gourgand et al. (2005), Dodin (1996), Honkomp et al. (1997), and Baker and Altheimer (2012), simulation-based techniques have been used to get results for the PFSPST. In most of

these articles, though, assumptions are made about the probability distributions employed to model processing times – e.g. Normal or Exponential – or about the restricted size of the instances being analysed. Nonetheless, in a real-life scenario, the specific distributions to be used will have to be fitted from historical data (observations) leading to empirical distributions. The assumption of processing times following a Normal distribution is, in our opinion, quite unrealistic and restrictive, since distributions such as the Log-normal or the Weibull are usually much better candidates to model processing times with positive values.

Thus, Peruyero et al. (2011) propose a simulation-optimization algorithm described next for solving the PFSPST. The main idea behind their approach is to transform the initial PFSPST instance into a PFSP instance and then to obtain a set of near-optimal solutions for the deterministic problem by using an efficient PFSP algorithm. The transformation step is achieved by simply considering the expected value of each stochastic processing time in the PFSPST as the constant processing time in the PFSP. Since any PFSP solution will be also a feasible PFSPST solution, it is possible to use Monte Carlo simulation to obtain estimates for the expected makespan. That is, it is possible to obtain these estimates by iteratively reproducing the stochastic behaviour of the processing times in the sequence of jobs given by the PFSP solution. Of course, this simulation process will take as many iterations as necessary to obtain accurate estimates. Simulation is used here to determine which solution, among the best-found deterministic ones, shows a lower expected makespan when considering stochastic times. This strategy assumes that a strong correlation exists between near-optimal solutions for the PFSP and near-optimal solutions for the PFSPST: Good solutions for the PFSP are likely to represent good solutions for the PFSPST. However, not necessarily the best-found PFSP solution will become the best-found PFSPST solution, since its resulting makespan might be quite sensitive to variations in the processing times. If there are indications for such as sensitivity, specific sensitivity studies could help to validate the results or at least to become aware of the dimension of uncertainly applied (cp. Rabe et al. 2010).

As the authors conclude, their approach offers a practical perspective which is able to deal with more realistic scenarios: by integrating Monte Carlo simulation in the methodology, it is possible to naturally consider any probabilistic distribution for modelling the random job processing times.

5   Conclusions and Outlook

In this paper, we have discussed how Monte-Carlo simulation can be combined with meta-heuristics in order to develop hybrid algorithms able to solve complex stochastic combinatorial optimization problems. The general scheme of these 'sim-heuristic' algorithms has been introduced, and some examples of applications to different fields have been described. These examples include stochastic versions of the well known inventory routing problem as well as the flow-shop problem. This work has focused on Monte-Carlo simulation, only. However, other types of simulation –e.g. discrete-event simulation– can be also hybridized with meta-heuristic approaches to solve complex optimization problems of stochastic nature in which the time factor must be also considered. As an example, current studies are extended to the problem of mid-term scheduling of production under relaxed constraints, e.g. with algorithms that allow for changing the factory's capabilities and capacities for the future. Such an

approach leads to so-called 'changing steady state' systems that show a high complexity with respect to the high number and diversity of relationships. DES seems a good candidate to approach this kind of problems in a combination with heuristic algorithms (cp. Rabe and Deininger 2013).

Acknowledgement

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