Joint Management of Material and Cash Flows

Gemeinsames Management von Material und Cash Flows

Enver Yücesan, INSEAD, Fontainebleau (France), enver.yucesan@insead.edu
Illana Bendavid, ORT Braude College of Engineering, Karmiel (Israel), illana@tx.technion.ac.il
Yale T. Herer, Israel Institute of Technology, Haifa (Israel), yale@technion.ac.il

Abstract: The objective of inventory management is to design effective policies for managing the trade-off between customer satisfaction and the cost of service. Over the years, these models have become increasingly sophisticated, incorporating many complicating factors that are relevant in practice. Curiously absent from these models are the financial constraints on the working capital. In this paper, we analyse the materials management practices of a self-financing firm whose replenishment decisions are constrained by cash flows driven by purchases and sales. Our study shows that arbitrarily imposed constraints on the working capital not only fail to prevent any violations, but also significantly distort operational decisions. We propose less myopic policies to eliminate these distortions.

1 Introduction

Working capital refers to the difference between current assets, which include cash, inventory and accounts receivable (A/R), and current liabilities, which include accounts payable (A/P) and short-term loans. The top 1000 European public companies have approximately 900 Billion Euros (9.4% of EU GDP) tied up in working capital (REL Consultancy 2012). Working capital management, whose objective is to increase the profitability of a firm by investing in long-term, high-return assets while ensuring sufficient liquidity during the operating cycle (Pass and Pike 1984), therefore represents a huge economic opportunity, not only for the economy as a whole, but for multitudes of individual firms as well. In particular, efficient working capital management is necessary to manage the inherent trade-off between profitability and liquidity since keeping too much cash for operations may lead to a smaller return from long-term investments.

Against this backdrop, our investigation into the management of inventory under working capital constraints was motivated by a distributor of industrial equipment that purchases heavy goods such as generators and earthmoving equipment from a manufacturer and sells them to various customers. The distributor grants trade credit
to its customers while receiving trade credit from the manufacturer. The latter, which has a finite production capacity, ships orders requiring a non-negligible delivery lead time and requires full payment after a fixed number of periods following delivery. This type of trade credit represents a one-part (net term) contract. Customer demand is random as it is driven by major construction projects. Once a delivery is made to a customer, the customer must pay for the invoice after a fixed number of periods. The distributor's challenge is to determine an inventory management policy that balances its inventory and shortage costs. With trade credits, inventory decisions have a direct impact on working capital levels since deferred inventory payments are recorded as A/P while delayed sales collection is captured as A/R. Typically, firms adopt a working capital policy to achieve the optimal trade-off between profitability of net current assets and the ability to pay for current liabilities under a clearly defined risk framework. Such a policy designates the portion of the sales revenue to be retained for working capital (Pass and Pike 1984). The inventory management problem thus becomes further complicated by the working capital policy. In this particular case, the distributor has an aggressive policy, choosing to keep a smaller portion of sales revenue as working capital. As a result, the working capital for the distributor cannot exceed a maximum value that is set by the distributor's parent company in an effort to limit inventory and collection risk. Throughout its operating cycle, which spans procurement, inventory holding, sales and cash collection, the distributor must therefore ensure adequate customer service in a self-financing fashion, that is, generating sufficient funds through sales to procure additional inventory. Note that this is in contrast with most of the recent work in the literature that focuses on payment default—hence, promotes a lower bound.

The recent spotlight on micro-retailing is another key motivator for our study. With little or no access to external financing, small, typically family-owned firms must carefully manage their operating cycle, generating sufficient cash from sales to replenish their inventories for business continuity. Pierson and van Ryzin (2010) describe such a micro-retail operation where the owning family needs to keep an eye on its working capital not only to effectively manage its operating cycle, but also to generate a sufficient cash surplus to make a living.

In this setting, cash flows generated by a firm’s operations have to be explicitly incorporated into the operational decision making process beyond the common practice of imposing an exogenous budget constraint on activities such as procurement, production, and sales. In the literature, however, until recently there has been little investigation of the relationship between finance and operations, and, in particular, of the interactions between material flows and cash flows. As a result, these decisions are often made separately without a unifying perspective on how these trade-offs affect the performance of the firm. This is also reflected in practice whereby a hard WCR constraint may be imposed on procurement while sales decisions are made without considering this constraint. The objective of this paper is therefore to construct a model to depict the interactions between material and cash flows as well as to quantify the impact of these interactions on performance. Through this study, we identify some unintended consequences of financial constraints; in particular, our study shows that arbitrarily imposed constraints on the working capital not only fail to prevent any undesired violations, but also sig-
Joint Management of Material and Cash Flows

Joint Management of Material and Cash Flows

nificantly distort operational decisions. We therefore propose an approach to eliminate these side effects.

2 Literature Review

Due to the fundamental Modigliani and Miller (1958) result in corporate finance theory whereby, in perfect and competitive markets, financing decisions can be made independently of investment and production decisions, there is traditionally little overlap between the extensive literature on cash management and the vast literature on production and inventory management. This separation is further reinforced in practice as the two functions normally reside in separate “silos” within the company, the former being part of treasury while the latter being part of operations. The realization that market imperfections such as accounting and bankruptcy costs, tax advantages of debt, and information asymmetry violate the key assumptions underlying the Modigliani-Miller result recently triggered a significant amount of work on the interaction between financial and operational decisions.

Dada and Hu (2008) consider the single-period inventory procurement problem of a capital-constrained newsvendor (CCNV). Given the procurement and selling price along with the distribution of demand, the CCNV determines the quantity that satisfies the newsvendor fractile, but may not have the financial means to order the optimal quantity. It must therefore determine how much to borrow at a given interest rate to finance additional procurement. In a Stackelberg game between the lender (the leader) and the CCNV (the follower), the authors show that even if the borrowing cost is not prohibitively high, the CCNV borrows funds to procure a quantity that is less than the channel optimal quantity. They further show that the lender charges an interest rate that decreases in CCNV's equity.

Li et al. (2013) characterize the optimal production, borrowing, and dividend decisions of a firm that aims to maximize the expected present value of the infinite-horizon flow of the dividends under uncertain demand, capacity, borrowing, and liquidity constraints.

Babich and Sobel (2004) focus on the coordination of financial and operational decisions to maximize the expected discounted proceeds from an initial public offering, modelled as a stopping time in an infinite-horizon Markov decision process.

Chao et al. (2008) analyse the replenishment decisions of a self-financing firm with the objective of maximizing its expected wealth at the end of a finite horizon. At the beginning of each period, the firm places an order with its cash on hand and deposits its surplus cash into a savings account to earn interest. During the period, demand is realized. At the end of the period, the firm collects cash generated from sales and interest earned from the savings account. Their model, however, assumes negligible replenishment lead times and does not consider the operating cycle, ignoring collection and payment periods.

In the presence of trade credits, Luo and Shang (2013) consider non-negligible payment and collection periods, and develop base stock policies that minimize the discounted total inventory and default costs over a finite horizon. When the collection period is longer than or equal to the payment period, they show the optimality of a myopic base stock policy. In the reverse setting, they propose a heuristic policy. We also examine the impact of trade credit on the operating cycle in
a working capital constrained environment where borrowing from a third party is not an option. Our analysis, however, differs from theirs in several aspects: first, we consider an infinite-horizon setting. Second, while we impose a hard constraint on the working capital, they consider a penalty for violating this constraint that reflects the risk of payment default. Finally, our model, which will be introduced in the next section, incorporates non-negligible replenishment lead times.

3 The Model

Consider a firm that procures goods from a manufacturer at a fixed purchase price to satisfy random demand. In particular, the firm faces an infinite-horizon newsvendor setting with backorders. In addition, the manufacturer has uncertain capacity that is independent of the order quantity. Mean demand is assumed to be smaller than mean capacity. The manufacturer delivers the product after a fixed order lead time. The firm has to pay for the received goods after a given payment period following delivery. When the firm sells a unit at a given selling price, the customer has to pay for the product after a fixed collection period. The goal is to minimize the long-run average cost of demand-supply mismatch while taking into account the uncertainty in customer demand and manufacturer capacity. In other words, the firm must determine an optimal inventory policy to minimize its (long-run average) cost for a specific combination of demand distribution, capacity distribution, order lead time, and cost parameters (holding and shortage costs). This challenge is further complicated by a constraint on the firm's working capital during its operating cycle. In this setting, two factors may limit the firm's ability to procure goods. First, the manufacturer may not be able to ship the requested replenishment quantity due its own capacity constraint. Second, due to the constraint on the firm's working capital, the firm may not have the funds to finance the totality of the desired replenishment quantity.

3.1 Notation and Problem Formulation

Throughout the paper, we will use the following notation:

**Random variables**

- $D_t$ is a random variable describing the I.I.D. demand in period $t$, with cumulative distribution function $F_{D_t}$, mean $\mu_D$, finite variance, and realization $d_t$.
- $K_t$ is a random variable describing the I.I.D. manufacturing capacity available in period $t$, with cumulative distribution function $F_{K_t}$, mean $\mu_K$, having a finite variance, and realization $k_t$.
- $R_t$ is a random variable describing the I.I.D. shortfalls/residuals at the end of period $t$, with cumulative distribution function $F_{R_t}$ and realization $r_t$.

**Problem parameters**

- $c$ is the per unit purchase price from the manufacturer.
- $L_o$ is the order delivery lead time from the manufacturer.
- $PP$ is the duration of the payment period, i.e., the number of periods during which the firm carries accounts payable (A/P).
- $V$ reflects the selling price per unit.
Joint Management of Material and Cash Flows

$CP$ is the duration of the collection period, i.e., the number of periods during which the firm carries accounts receivable (A/R).

$h$ is the holding cost per unit per period.

$p$ is the shortage cost per unit per period.

$CR$ represents the newsvendor critical ratio given by $p/(p + h)$.

$W$ is the maximum allowable value of the Working Capital Requirements (WCR), henceforth referred to as the WCR limit.

$\rho$ is the capacity utilization rate or the system load on the manufacturer, defined as $\mu_D / \mu_K$.

**Auxiliary and Decision Variables**

$S$ represents the order-up-to level.

$Q_t$ is the quantity ordered by the firm in period $t$ taking into account the order-up-to level, the available capacity, and the WCR limit.

$I_t$ is the net inventory at the end of period $t$.

$I_t^+$ represents the on-hand inventory at the end of period $t$ ($= \max\{0, I_t\}$).

$I_t^-$ captures the backorder level at the end of period $t$ ($= \max\{0, -I_t\}$).

$WCR_t$ reflects the requirements for working capital at the end of period $t$.

We wish to minimize the average system cost per period calculated as the sum of the cost of excess inventory and the cost of backorders, i.e.,

$$\frac{1}{n} \sum_{t=1}^{n} (hI_t^+ + pI_t^-),$$

Subject to the following constraints:

$$Q_t = \max\left\{0, \min\left\{S - I_t - \sum_{m=t-L_0+1}^{t-1} Q_m, K_t, \frac{W - WCR_t}{c}\right\}\right\} \text{ for all } t > 0$$

$$WCR_t = WCR_{t-1} + c \left(I_t^+ - I_{t-1}^+\right) + V \left(\min\{I_{t-1}^+, Q_{t-L_0}, D_t + I_{t-1}^-\} - \min\{I_{t-1-CP}, Q_{t-CP-L_0}, D_{t-CP} + I_{t-1-CP}^-\}\right) - c \left(Q_{t-L_0} - Q_{t-PP-L_0}\right) \text{ for all } t > 0$$

$$I_t = I_t^+ = I_t^- = 0 \text{ for all } t \leq 0$$

$$WCR_t = Q_t = 0 \text{ for all } t \leq 0$$

The first equation defines the quantity ordered in period $t$, as the minimum of the quantity needed to attain the order-up-to level $S$, the realized manufacturer capacity, and the quantity corresponding to the monetary value of the working capital.
available to the firm. The last element of this minimum highlights the difference between \( W \) and \( WCR_t \), where the former is the amount of wealth to which the firm has access and the latter is the amount of this wealth that the firm is using at time \( t \).

The next equation updates the \( WCR \), where we add the change in the monetary value of the inventory and the change to the A/R, adding the value of sales in period \( t \) and subtracting the value of sales in period \( t - CP \). Finally, we subtract the change in the A/P, the difference in the value of items delivered in period \( t \) and the items delivered in period \( t - PP \). The last two equations initialize the system with boundary conditions. Note that the optimal order-up-to level, \( S \), is computed numerically following the approach described in Güllü (1998). We refer to this formulation as the Base Model.

Unfortunately, an arbitrarily imposed constraint on the working capital not only fails to prevent any undesired WCR violations, but also significantly distorts operational decisions. This phenomenon is driven by the fact that while a hard constraint on working capital is strictly enforced in procurement, sales decisions are typically made without taking it into consideration. To eliminate these distortions, instead of limiting our order quantity by the current WCR value, we order no more than the quantity, which ensures that the probability of violating the constraint on the working capital in the future, due to this order, will be less than or equal to \( \beta \) (0 < \( \beta \) < 1), where \( \beta \) is a model parameter that reflects the firm’s risk appetite. More specifically,

\[
Q_t = \min \left\{ S - I_t - \sum_{m=1}^{t-1} Q_{m}, K_t, \arg \max \right\} \left\{ \text{Prob} \left( \max_{\tau:t+LO} wcr_{\tau}(Q) > W \right) \leq \beta \right\} \\
\text{for all } t > 0
\]

Since the only two time epochs at which WCR increases are when we pay for the items we ordered (i.e., \( LO + PP \) time units after placing the order) and when we sell an item, it suffices to evaluate \( wcr_{\tau} \) for \( \tau \) from \( t + LO \) up to \( t + LO + PP \), or whenever the inventory runs out, whichever occurs later. We begin the evaluation at time \( t + LO \) because this is the first time the effect of an order placed at time \( t \) is felt. To evaluate the term, we use simulation. That is, we independently replicate the system starting in period \( t \) and, for each replication, we find the maximal order quantity in period \( t \) that does not violate the constraint on the working capital.

The manufacturer then ships the minimum of the replenishment quantity ordered by the firm and its realized capacity. Let us refer to this modified version as the Spend-at-Risk Model.

### 3.2 Observations

As anticipated, the system with no constraints on the working capital has the lowest average cost (AC) and the highest percentage of violations (PV). The effect of \( \beta \) was also consistent with our expectations. Lower values of \( \beta \) (tighter constraints) result in higher AC and lower PV values. Of the highest interest is the comparison of the Base Model with the Spend-at-Risk Model. The AC of the Base Model is higher than even the most constrained Spend-at-Risk Model (\( \beta = 0 \)). In and of itself, this result is not surprising; one would expect a trade-off between AC and PV. However, combined with the fact that the Base Model had a higher PV value than...
the Spend-at-Risk Model for both $\beta = 0$ and $\beta = 0.25$, this finding means that both of these policies dominate the Base Model. This is best illustrated graphically in Figure 1, where we plot the performance of various policies based on their 90th percentile AC and PV values. Note that the Base Model does not even lie on the efficient frontier.

Figure 1: Average cost versus WCR violations

Having observed that the Spend-at-Risk Model is superior to the Base Model, we now examine its behavior. We start by focusing on the newsvendor parameters. For $\beta = 0$, there is no qualitative difference in the impact these parameters have on the AC between the Base Model and the Spend-at-Risk Model. The less myopic perspective of the latter model, however, is noticeable in the percentage of time the order quantity is constrained by WCR, PM, and PV. For instance, a higher $CR$ leads to a higher PM 41% of the time in the Base Model and 30% of the time in the Spend-at-Risk Model. Similarly, a longer $LO$ leads to a higher PV 22% (14%) of the time in the Base (Spend-at-Risk) Model.

With respect to the financial parameters, the AC is significantly lower in the Spend-at-Risk Model. An increase in $V$ and $CP$ leads to an increase in the AC in a smaller percentage of the cases compared with the Base Model. Notable is the behavior of the parameter $c$. Whereas in the Base Model an increase in the unit purchase price causes an increase in AC in more cases than it causes a decrease (21% versus 16%), this trend is reversed in the Spend-at-Risk Model where AC decreases in more cases than it increases (14% versus 12%). The reason for this switch lies in how this parameter is treated in the two models. While the Base Model incorporates this parameter in three ways (accounts payable, inventory evaluation, and directly in the ordering decision), the Spend-at-Risk Model incorporates the parameter only in two ways (accounts payable and inventory evaluation). In other words, an increase in $c$ could very well force the Base Model to order less, i.e., further from the average cost minimizing order-up-to level. There is, however, no such effect in the Spend-at-Risk Model. We also note that the ‘lethal’ interaction between $V$ and $CP$, which consumes a larger portion of WCR for a longer period of time, has a more mitigated impact on both AC and PM under the Spend-at-Risk Model. We can therefore conclude that
this model not only eliminates this instability, but also lowers the average total cost per period. Unfortunately, it does not eliminate all the violations on the working capital constraint.

4 Conclusions

The objective of inventory management models is to determine effective policies for managing the trade-off between customer satisfaction and the cost of service. Over the years, these models have become increasingly sophisticated, incorporating many complicating factors that are relevant in practice such as demand uncertainty, finite supplier capacity, and yield losses. Curiously absent from these models are the financial constraints imposed by working capital requirements (WCR). In practice, however, many firms are self-financing, i.e., their ability to replenish their own inventories is directly affected not only by their current inventory levels, but also by their receivables (trade credit they have extended to their customers) and payables (trade credit they have received from their supplier). Such constraints have gained added significance during the lingering economic crisis, which has made external financing increasingly difficult to secure. In this paper, we analyzed the materials management practices of a self-financing firm whose replenishment decisions are constrained by cash flows, which are updated periodically following purchases and sales in each period. In particular, we investigated the interaction between the financial and operational parameters as well as the impact of the constraints on the working capital on the long-run average cost.

The key messages of our study can be summarized as follows:
1. The imposition of a myopic constraint on WCR renders an otherwise stable operating cycle unstable by leading to shortfalls that become increasingly difficult to eliminate.
2. An arbitrarily imposed constraint on the working capital not only fails to prevent any undesired violations, but also significantly distorts operational decisions. This phenomenon is driven by the fact that while a hard constraint on working capital may be strictly enforced in procurement, sales decisions are typically made without considering the constraint.
3. The unintended consequences of the myopic WCR constraint mentioned before lead us to define a more flexible and forward-looking approach, which is implemented in the form of a spend-at-risk model. This model not only eliminates this instability, but also lowers the average total cost per period. Unfortunately, it does not eliminate all the violations on the working capital constraints.

References


