

# **A Location Model for Dynamic Vehicle Routing Problems**

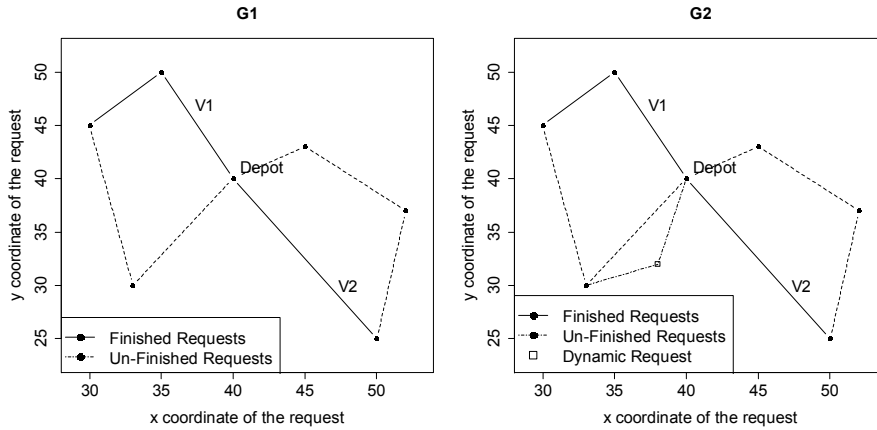
## ***Ein Ortsmodell für dynamische Fahrzeugwegesucheprobleme***

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**Abstract:** Multi-constrained Vehicle Routing Problems (VRP) are gaining in interest. Especially the dynamic version of the problem gets more emphasis, due to modern service requirements, like short-term or express delivery. Current models reflect the dynamism of a problem by simply considering the number of dynamic requests. A more feature-rich model for the dynamism in Dynamic Vehicle Routing Problems (DVRP) would allow an improved design and a better comparison of optimization algorithms. Additionally, a more problem instance specific algorithm selection would be possible. This paper introduces a model for the location of dynamic requests in a DVRP, the Location-based Degree of Dynamism (LDOD). Each dynamic request in every problem variant has inherently a location. Our experiments show a positive correlation between the proposed LDOD and the resulting DVRP solution quality. We conclude the paper with a general discussion and a broad outlook on our research.

## **1 Introduction and Related Work**

We order goods over the internet and our daily groceries are hopefully delivered fast and freshly to our doorstep. We share cars and use various transportation systems. We plan our trips with navigation applications and expect what we need to be in stock in the nearest supermarket. To provide all these services, complex Vehicle Routing Problems (VRPs) with several constraints have to be managed and solved. The VRP was first introduced by Dantzig and Ramser (1959) and has been studied extensively over decades. Due to the expansion of short-time and express services, especially the dynamic version of the problem is gaining in interest. If not all customer requests are known in advance, the problem is defined as a Dynamic Vehicle Routing Problem (DVRP), see, for example, the taxonomy introduced in Lahyani et al. (2015). Figure 1 illustrates an example of a dynamic vehicle routing. The DVRP is first introduced in Wilson and Colvin (1977) as an extension of the VRP, with a description of a computer controlled Dial-A-Ride system in Rochester, NY (USA). The software handling this demand-oriented public transportation system has to schedule customer requests to available vehicles continuously during the day.



**Figure 1:** Graph G1 shows a visualization of a VRP at a time  $t_i > 0$ . In G2 a dynamic request occurs at time  $t_i$  and planned routes have to be reorganized to satisfy the additional request.

Since the late 70s, considerable research has considered the dynamism in vehicle routing. Pillac et al. (2013), and Psaraftis et al. (2016) describe an explosion in related papers after the year 2000. Despite the intense research in this area, Ritzinger et al. (2016) describe a research gap with respect to comparability of results and employed approaches. Our long term research intention is to participate in closing this gap by developing a DVRP model instance generator to support the evaluation of algorithms solving the DVRP with simulation approaches. Generated model instances could be executed for example with the open-source discrete event simulator for rich vehicle routing problems (RVRP Simulator) introduced in Mayer et al. (2016). We would like to establish simulation as a methodology to investigate, evaluate, and solve DVRPs. To this end, we need to develop a deep understanding of the nature of dynamic requests in routing problems to be able to generate and compare instances, and respectively to be able to develop and compare solution approaches.

### 1.1 Dynamism in a Routing System

Early work capturing the dynamic effects in a routing system was done by Lund et al. (1996). They introduce the Degree of Dynamism (DOD) as shown in Equation 1, a measure of the dynamism of a routing problem.

$$DOD = \frac{n_{imm}}{n_{tot}} \quad (1)$$

The DOD describes the ratio between dynamic requests,  $n_{imm}$ , and total requests  $n_{tot}$ , where the total requests are the sum of dynamic and static requests. Larsen (2000) extended the DOD by including the aspect of time. He introduced the Effective Degree of Dynamism (EDOD) as shown in Equation 2, which considers the planning horizon T for the calculation of the measure of the dynamism of a routing problem. The EDOD allows the modelling of the dynamic arrival times.

$$EDOD = \frac{\sum_{i=1}^{n_{imm}} t_i}{n_{tot} T} \quad (2)$$

The EDOD allows the modelling of the dynamic arrival times. Larsen (2000) also introduces the Effective Degree of Dynamism with Time Windows (EDOD-TW), which considers the reaction time  $r_i$ . The reaction time is defined as time between the occurrence of the dynamic request  $t_i$  and the latest possible time the service should begin,  $l_i$ . As discussed before, the number and the arrival time of dynamic requests within the planning horizon is essential to describe the dynamism of a routing system (Larsen 2000). However, Mendoza et al. (2014) consider amongst others the parameters shown in Table 1 for a static request in their open-source VRP model, named VRP REP Model. We assume that the mentioned parameters should also be considered for the description of the dynamism in a routing system that has any dynamic requests. First considerations in this direction are given by Larsen et al. (2007). They suggest that future research should also expand the degree of dynamism measure by including the service time and the demand size.

**Table 1:** Extraction of parameters with description, defined for the static request of a VRP in the VRP REP Model from Mendoza et al. (2014). Dynamic requests are not part of the VRP REP Model. Additional Parameters to be considered by the description of the dynamism are also listed.

Parameters from VRP REP Model	
Node	The location of the request.
Service Time	The expected service time
Quantity	The amount/capacity/size of the demand
Skill	Skills needed to handle the request
Resource	Resources needed to handle the request
Dimension	The dimension of the demand
Additional Parameters to consider	
Weight	The weight of the demand

The parameters listed in Table 1 are mostly problem instance specific. Most of the studied VRP instances, for example, do not consider skills or resources needed to service a certain request. The dimension and weight of transported goods are often neglected, too. Independently from more specific problem instance characteristics, each dynamic request has at least a given time and location. This property is summarized in the following lemma.

*Lemma of minimum characteristics of a dynamic request in a DVRP: A dynamic event in a DVRP has at least the characteristics time and location.*

We assume that in addition to the amount of dynamic requests both described inherent characteristics of a dynamic request influence the solution quality of a DVRP. For the static VRP with no capacity constraints, the Travelling Salesman Problem (TSP), the variance of the distance matrix correlates with the difficulty of the problem

(Cheeseman et al. 1991; Ridge and Kudenko 2007). Influences of other TSP meta-data for the problem difficulty is investigated in Smith-Miles et al. (2010).

The number of dynamic requests is modelled with the help of the DOD; the time of occurrence is described with the EDOD. But as far as we know, there is currently no model of the third most basic characteristic of a dynamic request, the location where the request occurs in the context of a DVRP. Therefore, this paper focuses on the location of a dynamic request. In Section 2 we introduce a location model and show how our location model is correlated to the DOD. Section 3 discusses the experimental design by introducing the DVRP instance generation and solving process considering the introduced lemma. In Section 4 we present the results of our experiments and we show that there is a statistically significant positive correlation between the solution quality and our location model. Our paper concludes with a discussion of our results, and the impacts of time and employed heuristics on the solution quality. We also provide an outlook of our future research.

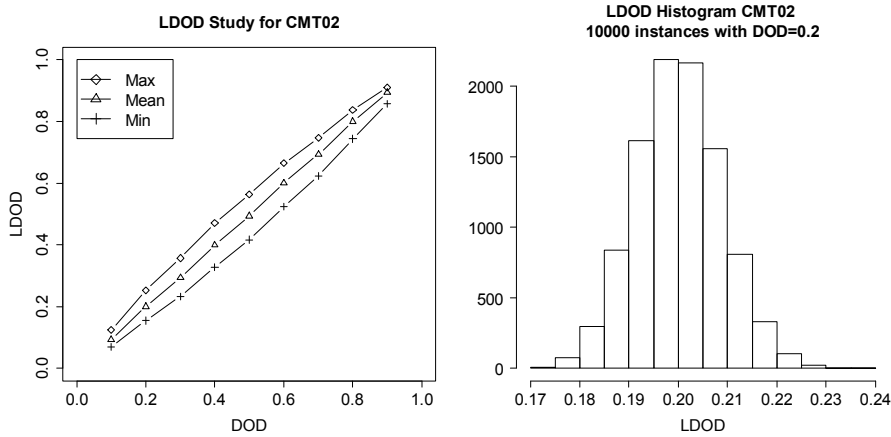
## 2 The Location-based Degree of Dynamism

We introduce the Location-based Degree of Dynamism (LDOD) as a model for the locations of the dynamic requests in the context of a DVRP. The calculation of the degree is inspired by the data-mining algorithm called Density-Based Spatial Clustering of Applications with Noise (DBSCAN), introduced in Ester et al. (1996). This algorithm clusters points by their density and separates them into the categories core, reachable, and not reachable. The calculation of the LDOD is shown in Equation 3, in which the function *distance* calculates the distance between the two points  $P_i$  and  $P_k$ .

$$LDOD = \frac{\sum_{i=1}^{n_{imm}} \sum_{k=1}^{n_{tot}} distance(P_i, P_k)}{\sum_{i=1}^{n_{tot}} \sum_{k=1}^{n_{tot}} distance(P_i, P_k)} \quad (3)$$

The scope of the LDOD is as follows:  $0 \leq LDOD \leq 1$ . A LDOD of 0 can only be achieved if the sum of distances between points of dynamic requests has no proportion of the overall distance between all points. This is only possible if there are no dynamic requests, meaning the DOD is 0. If the DOD is 1, all requests are dynamic requests, so the LDOD is 1 too. So the LDOD is highly dependent on the DOD. The relation between DOD and LDOD is visualized in Figure 2.

The diagram on the left shows the possible LDOD value range for a given DOD on the VRP instance CMT02 introduced in Christofides et al. (1979). The diagram on the right shows the histogram of the LDOD values for 10000 randomly generated instances with a DOD of 0.2. The diagrams show that the LDOD correlates with the DOD but there are instances with equal DOD and different LDOD. We investigated all VRP instances introduced in Christofides et al. (1979), Solomon (1987), and Uchoa et al. (2014). All instances exhibit a similar relation between LDOD and DOD.



**Figure 2:** Relation between DOD and LDOD for the VRP instance CMT02 introduced in Christofides et al. (1979).

### 3 Design of Experiments

For the investigation of a possible correlation between our newly introduced LDOD, a model for the locations of dynamic requests in a DVRP and the problem solution quality, we need to create and solve DVRP instances. For this paper, we equate the solution quality with the costs of a solution which is equal to the travelled distance. Due to the lack of DVRP instances, we generated them from static VRP instances introduced in Christofides et al. (1979). We reduced these instances to TSP instances by eliminating vehicle capacity, requested quantity, and service time. Note, that we do not introduce additional requests into the problem, but instead transform some static requests into dynamic requests. The amount of dynamic requests is determined by the DOD (by default we use a value of 0.2). The VRP instance CMT13 from Christofides et al. (1979) models the most requests (120). So, for CMT13, with a DOD of 0.2, the possible number of different DVRP instances is  $1.0872202 E25$  ( $n_{imm} = 24$  out of  $n_{tot} = 120$ ). We identify the instances with the minimum possible and the maximum possible LDOD. Additionally, we generate 48 randomly selected instances.

For the representation of the generated DVRP instances, we extended the VRP-REP format introduced in Mendoza et al. (2014) by including labels for dynamic requests. A DVRP consists of a set of static ( $n_{tot} - n_{imm}$ ) and a set of dynamic ( $n_{imm}$ ) requests. The routing solution for all static requests is generated with the help of the Jsprit framework, one of the most popular and powerful open-source Java frameworks for creating and solving VRP's introduced and maintained by (Schröder 2014). The determined costs for the initial solution defines the time horizon T to calculate the arrival times of the dynamic requests using the EDOD. The EDOD has a value of 0.2. The calculated arrival times  $t_i$  are evenly distributed between 0 and T to minimize any influences of the arrival times on the dynamic requests, see Equation 4. The resulting DVRP model in the VRP-REP format and the routing solution for all static requests are transformed into the RVRP Simulation Model introduced in (Mayer et al. 2016).

$$t_{i+1} - t_i = d_i; d_i = d_{i+1} \forall \text{ Request } i \quad (4)$$

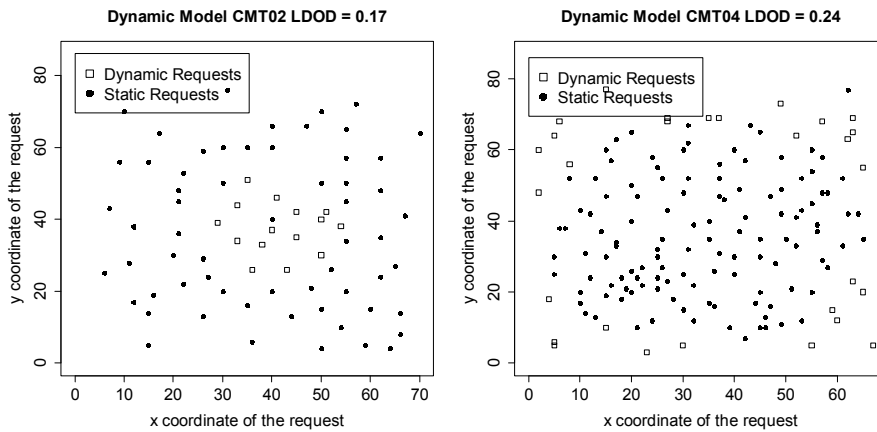
The corresponding RVRP Simulator from Mayer et al. (2016) offers an interface for handling dynamic requests during the simulation. We implemented the two following dispatching algorithms.

1. Greedy: Every occurring dynamic request is added to the existing tour in a greedy way. We try to place the new request between two successive locations in the planned tour. The greedy algorithm selects the placement that leads to the minimal increase of costs. If no current tour exists, i.e., the planned tour is already completed, a new tour is generated.
2. Jsprit: Every occurring dynamic request triggers a complete re-planning of the existing tour by means of Jsprit. If no current tour exists a new tour is generated.

The experiments are performed with all 14 VRP instances introduced in Christofides et al. (1979) with both dynamic requests handling algorithms.

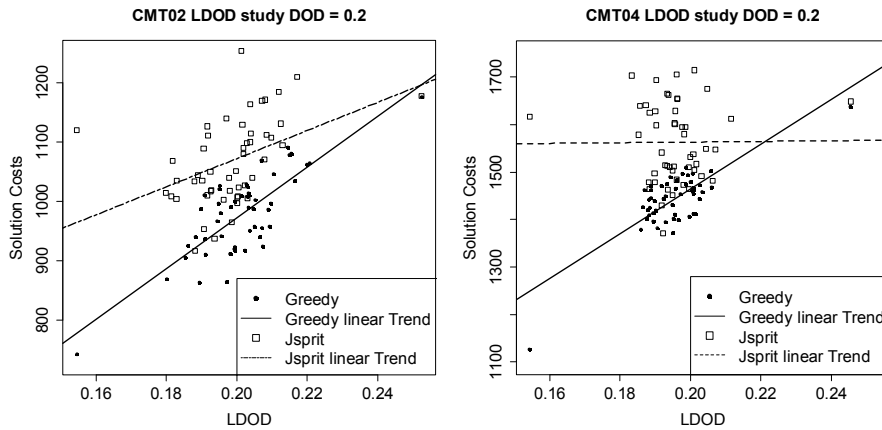
## 4 Results of Experiments

In this Section, we provide an overview on our experimental results. Example DVRP instances, generated from static instances are shown in Figure 3.



**Figure 3:** Generated DVRP instances. The left instance is based on CMT02 with a low LDOD. A low LDOD leads to dynamic requests within the core, compare to DBSCAN introduced in Section 2. The right instance is based on CMT04 with a high LDOD. A high LDOD leads to dynamic requests within categories reachable or not reachable, compare to DBSCAN.

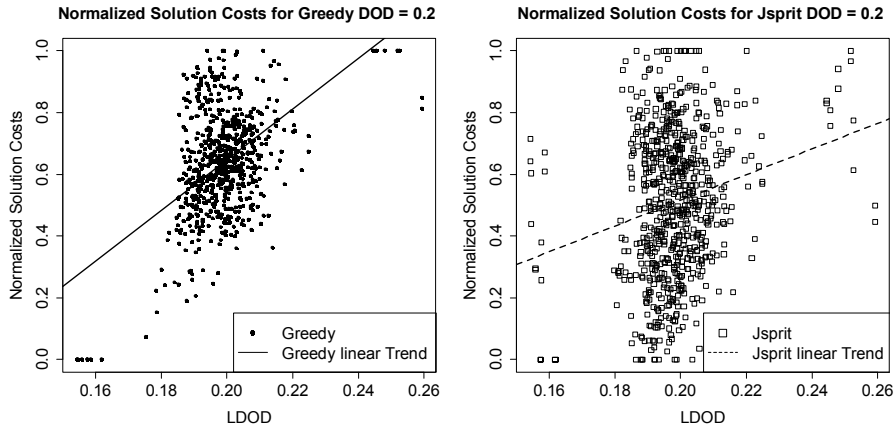
Figure 3 illustrates the effect of different LDODs for a constant DOD. A low LDOD leads to dynamic requests within the category core, compare to DBSCAN introduced in Section 2. A high LDOD will pick locations from the category reachable and not reachable for the dynamic requests. Figure 4 shows an overview of the results for the instances CMT02 and CMT04. Each graph displays the relation between the solution costs on the y-axis and the LDOD on the x-axis for both implemented dispatching algorithms.



**Figure 4:** Visualisation of the correlation between solution costs and LDOD for the instances CMT02 and CMT04. Solution costs are determined for both instances with the Greedy and Jsprit algorithm.

Since we picked 48 dynamic realizations of the instances randomly, the values of the LDOD are varying around the value of the DOD. The calculated Spearman's rank correlation, introduced from Spearman (1904), shows a statistically significant positive correlation between the LDOD and the solution costs for close to all instances for both algorithms as indicated from the linear trend line also shown in Figure 4. For the instance CMT02, the Spearman correlation value is 0.52 for the solution with Jsprit and 0.63 for the Greedy solution. Both values are statistically significant with a p-value of less than 0.05. For the instance CMT04 the correlation value for the Greedy solutions is 0.48, also statistically significant with a p-value of less than 0.05. The Jsprit solution for the instance CMT04 is not correlated and therefore an exception among all instances. The results of this experiment are not showing any correlation although the instances are quite similar, see Figure 3.

A representation of the results for all instances is shown in Figure 5 where we normalized the solution costs for all instances. The minimum solution costs for an instance is represented by the value 0, and the highest costs are represented by the value 1. The accumulated results over all instances show a statistically significant positive correlation between LDOD and costs, independently of the algorithm. The Spearman correlation value is 0.30 for the normalized values of the costs generated by the Greedy algorithm. The correlation value for the normalized solution costs generated by Jsprit is 0.15. The results for all instances introduced in Solomon (1987) are also showing a statistically significant positive correlation between LDOD and costs, determined with the greedy algorithm. The Spearman correlation value is 0.24 for these instances. Due to time constraints, we were not able to determine cost values with the Jsprit algorithm for these instances.



**Figure 5:** Visualization of the correlation between the normalized solution costs and the LDOD for all evaluated instances.

## 5 Discussion

In general, our research shows that there is a correlation between the solution costs of a DVRP and the locations of the dynamic requests. It seems that there is a difference in solving the problem if the dynamic requests occur close to each other, in the core area, or at the margins of the problem location space. We currently imply that higher costs are the result of a more difficult problem instance for the investigated algorithms. But maybe other algorithms behave differently. Macready and Wolpert (1996) postulate that problem instance specific characteristics determine the difficulty of an optimization problem for a particular algorithm (Smith-Miles et al. 2010). In our future research, we will investigate the correlation between solution costs and the locations of the dynamic requests for other DVRP solution approaches like Waiting & Relocation Strategies, or Dynamic Programming (Psaraftis et al. 2016), too.

Our introduced LDOD is currently not considering the variance in the location matrix of the dynamic requests and has a strong relation to the DOD. We are currently working on a location model which is independent from the DOD and which considers the variance. We think, our new model will lead to an even stronger positive correlation between the new model and the solution costs.

Our experiments also show that the solution costs are influenced by the time horizon  $T$  needed for the calculation of the EDOD, as shown in Equation 2. We additionally executed our experiments with  $T_n = T * 1,5$  which lead to higher solution costs compared to the presented results with  $T_n = T$ . The parameter  $T$  defines the latest possible time a dynamic request can occur. We determined the parameter using the time needed to satisfy all static requests. We suspect that the parameter  $T$  is actually quite problem instance specific. We can imagine different company strategies. One strategy, for example, which only accepts dynamic requests until a certain point of time during the day. So, if all static requests are satisfied during the whole day our estimation for  $T$  is too high. On the other hand, if the static requests can be satisfied earlier during the day because dynamic requests are expected our estimation for  $T$  is too low.



Due to the time and resources consuming experiments, we only investigated the relation between solution costs and location model with a DOD value of 0.2. We expect to see also a positive correlation between location model and solution quality for higher DOD but for a final clarification we also have to perform these experiments.

What we also see is that the performance of Jsprit is worse than the performance of the Greedy algorithm. Every time a dynamic request occurs, Jsprit plans a new route beginning and ending at the current location. We were not able to configure Jsprit to not plan a complete round trip due to an unresolved Jsprit issue. This may lead to the observed performance differences. On the other side, a complete reorganization of the route at any time a dynamic request occurs may not always be the best idea in general. The complete reorganization could destroy promising routes due to changing of the direction for example. Our future investigation of models for the dynamism of DVRPs will include other algorithms as well.

## 6 Conclusion and Outlook

Our research shows that there is a significant positive correlation between our newly introduced LDOD, a model for the location of dynamic requests in a DVRP, and the solution quality for a constant DOD. The positive correlation is demonstrated for different DVRP solution algorithms and for different VRP instances. That means, that for example a comparison of DVRP algorithms only based on the DOD is not sufficient and could lead to wrong evaluations.

Our future research focuses on the improvement of our newly introduced LDOD by considering the variance within the dynamic request location matrix. We will also concentrate on the development of models describing other characteristics of a dynamic request influencing the dynamism of a DVRP. Our long term research goal is the generalization of the models of the dynamism for DVRP to provide high quality problem instances and to support the DVRP solution algorithm development. We think by improving the modelling of the characteristics of the dynamism in a DVRP we are able to design better solution algorithms.

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